Minimum Entropy Principle

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The minimum entropy principle is a consequence on the second law of thermodynamics that, in essence, says that the entropy of a material particle in a flow field is constant along its pathline except that it increases upon crossing a shock wave, viscous layer, or thermal conduction region. In numerical simulations involving shock waves, to avoid truly discontinuous solutions often an artificial viscosity (with a subsequent continuous, smeared solution profile) is added locally in the vicinity of the shock wave to replace the physical effect of the entropy production of the shock wave with the equivalent entropy increase of a viscous layer. Ideally this will give the correct energy partitioning between kinetic and internal energy. However if the mesh spacing does not permit sufficient velocity gradient for the given shock jump solution or the viscous coefficient is not set appropriately the resulting entropy production will not be equivalent to that of the true shock. If the numerical method is conservative, it may even create a train of wiggles in the solution, radiating away from the shock/viscous layer, whose gradients will indeed attempt to produce an entropy production equivalent to that of the shock jump. The problem is that this smeared shock with additional spatial wiggles is not the good solution ones desires. So it is the proper setting of the viscosity coefficient along with the jump conditions for the given mesh size which is desired; to produce a monotonic increase through the viscous layer with as little smearing as possible, but with entropy production matching that of the true shock.

Tadmor [appl. num. math. 2 (1986) 211-219] gives an alternative statement (more useful for constructing numerical methods and proofs) of the entropy minimum principle by transforming the Lagrangian statement above into that of an Eulerian reference frame. Further discussion may be found in Kroner, LeFloch, and Thanh [esaim: math. modeling and num. analy. 42 (2008) 425-442] and in Guermond and Popov [siam: j. appl. math. 74 (2014) 284-305]. A more physical introduction, with use of the second law of thermodynamics to construct compatible constitutive relations, may be found in Bowen [Intro. to Continuum Mech. for Engineers, Plenum Press, New York, 1989].